ESTIMATING POTENTIAL OUTPUT, CORE INFLATION AND THE NAIRU AS LATENT VARIABLES

*Rafael Doménech**
*Víctor Gómez***

SGAPE-2003-06

September 2003

* University of Valencia and Ministerio de Hacienda.
** Ministerio de Hacienda.

Address for comments:
Victor Gómez Dirección Gral. de Presupuestos, Subdirección Gral. de Análisis y Programación Económica., Alberto Alcocer 2, 1-P, D-34, 28046, Madrid, Spain.(*vgomez@sepg.minhac.es*)

This paper has benefited from the valuable suggestions of J. Andrés and F. Corrales, and the participants at different seminars. R. Doménech acknowledges the financial support of CICYT SEC2002-266.

Abstract

This paper proposes a new method to obtain estimates of the NAIRU, the core inflation and the investment rate trend for the United States using an unobserved components model which is compatible with the usual decomposition of real gross domestic product into trend and cycle. The model includes a standard Okun's law, a forward-looking Phillips curve and an investment equation. The unknown parameters in the model are estimated by maximum likelihood using a Kalman filter initialized with a partially diffuse prior, and the unobserved components are estimated using a smoothing algorithm. Our results show that the output gap is positively correlated with the deviations of the investment rate from its trend and the inflation rate from core inflation, and negatively correlated with the deviations of the unemployment rate from the NAIRU.

KEY WORDS: Output gap, forward-looking Phillips curve, Okun's law, investment, Kalman filter.

1 Introduction

A useful decomposition of output into its trend and cyclical components should account for three central stylized facts in modern macroeconomics:

1. The negative correlation between the deviation of output from its trend and the deviation of the unemployment rate from the structural rate of unemployment, or NAIRU (non–accelerating inflation rate of unemployment) as it is sometimes called. This relationship between the cyclical components of output and unemployment is a manifestation of the well-known Okun's Law.

2. The trade-off in the short run between inflation and unemployment, which leads Mankiw (2001) to assert that "it is impossible to make sense of the business cycle ... unless we admit the existence of such a trade-off".

3. The comovement of output and investment. This is one of the most important regularities of business cycles, independently of the detrending method (Stadler, 1994, Canova, 1998, Burnside, 1998). Since investment is more volatile than the gross domestic product (GDP), the investment rate increases in expansions and falls in recessions.

Taking these facts together, it seems that the unemployment, inflation and investment rates contain very important information about the cyclical position of the economy and, therefore, of the output gap. In this paper, we take all this evidence into consideration and propose an unobserved components model for the United States which will allow us to obtain time-varying estimates of the NAIRU, core inflation and the structural investment rate which are compatible with the usual decomposition of the GDP into trend and cycle. The different cyclical components in the model are specified in terms of the output gap. The model is estimated by maximum likelihood through the use of the Kalman filter initialized with a partially diffuse prior. A smoothing algorithm is used to obtain estimates of the unobserved components based on the whole sample together with their mean squared errors.

In contrast to our approach, previous research trying to obtain alternative estimates of the output gap for the US or the European countries has omitted at least one of the three facts mentioned above. For example, Kuttner (1994) uses only the information contained in inflation through a simple backward-looking version of the Phillips curve. Apel and Jansson (1999) and Camba-Méndez and Palenzuela (2003) do not consider the investment rate and their estimated Phillips curve does not include any time-varying component which proxies core or expected inflation. Alternatively, Gerlach and Smets (1999) consider only a backward-looking Phillips curve and an aggregate demand equation which relates the output gap to its own lags and the real interest rate. Laubach (2001) has proposed a model consisting only of a Phillips curve linking the first difference of inflation to cyclical unemployment and the equations necessary to model the two unobservable components (the NAIRU and the gap) of the unemployment rate. Their model is similar to the one proposed by Gordon (1997), but allowing the NAIRU to be a non-stationary process in some countries. Using a similar framework, Staigner, Stock and Watson (2001) take advantage of the information contained in the inflation rate and the growth of real wages to compute a time-varying estimate of the NAIRU. Roberts (2001a) decomposes output into labor productivity and hours, obtaining the trend and cyclical components using the additional information of the inflation rate through the estimation of a backward-looking version of the Phillips curve. More recently, Rünstler (2002) estimates the real-time output gap in a supply curve, but he further investigates alternative extensions including the unemployment rate, capital stock, productivity and capacity utilization.

In short, to the best of our knowledge, previous research has made no use of the rich information about the business cycle simultaneously contained in the GDP and the unemployment, inflation and investment rates to obtain a better decomposition of these four variables into trend and cyclical movements. However, our results show that the output gap estimated as a latent variable is very significant in the three equations that we use to specify the relationships among the previous four variables, namely, the Okun's law, a forward-looking Phillips curve and an accelerator-type investment equation.

Besides the contribution in terms of the model specification to include additional information from relevant macroeconomic variables, we use a very flexible methodology that has a solid foundation and is specially designed for nonstationary state-space models, where the initial conditions for the Kalman filter are not well defined. Specifically, the initial state vector is modelled as partially diffuse and a "diffuse Kalman filter" (De Jong, 1991) is used for prediction and likelihood evaluation. At a later stage, we use a smoothing algorithm to obtain estimates of the unobserved components together with their confidence intervals. It is not unusual in the previous context to find that no two authors approach the initialization problem in the Kalman filter the same way. More often than not the model assumptions are not elucidated, the initial state is not explicitly defined, and the initial conditions for the Kalman filter are obtained by using some approximation such as a backcasting device. This causes problems in the optimization routine and many parameters have to fixed to some pre–specified, and somewhat arbitrary, values.

The paper is structured as follows. In section two we present the unobserved components model used to decompose each variable into a trend and a cyclical component, and we discuss some estimation issues. The third section presents the results of the estimation of our model and some basic features of the estimated unobserved components. In order to evaluate the validity of our decomposition, the fourth section analyzes some properties of our estimates in terms of revisions and inflation forecasts, compared with alternative procedures. Finally, section four summarizes the conclusions.

2 The potential output model

2.1 Output decomposition

We begin by decomposing the log of real GDP, y_t, into trend, $\overline{y}_t$, and cyclical output, y_t^c:

$$y_t \equiv \overline{y}_t + y_t^c. \tag{1}$$

The trend component is assumed to follow a sufficiently general process such that its rate of growth is either a stationary process or a random walk

$$\begin{aligned} \triangle \overline{y}_t &= \gamma_{yt} \\ \gamma_{yt} &= (1-\rho_y)\overline{\gamma}_y + \rho_y \gamma_{yt-1} + \omega_{\gamma t}, \end{aligned}$$

where $\triangle = 1 - L$, L is the lag operator, $L\overline{y}_t = \overline{y}_{t-1}$, $0 \leq \rho_y \leq 1$, and $\omega_{\gamma t}$ is assumed to be an i.i.d. $N(0, \sigma^2_{\gamma\omega})$ sequence. If $\rho_y = 1$, then $\triangle\overline{y}_t$ is $I(1)$ and y_t is $I(2)$, where $I(1)$ and $I(2)$ refer to integrated processes of order one and two. On the contrary, if $\rho_y < 1$, $\triangle\overline{y}_t$ is $I(0)$ and y_t is $I(1)$.

As for the cyclical component, we make the assumption that it follows a stationary $AR(2)$ process with complex roots

$$y_t^c = 2\theta_1 \cos(\theta_2) y_{t-1}^c - \theta_1^2 y_{t-2}^c + \omega_{yt},$$

where ω_{yt} is assumed to be an i.i.d. $N(0, \sigma^2_{y\omega})$ sequence and $0 < \theta_1 < 1$.

A useful insight into our proposal can be obtained by comparing our specification with that of the Hodrick and Prescott (1997) filter (henceforth HP filter). In the model-based interpretation of this filter (Gómez, 1999), output is also expressed as in (1), but the rate of growth of the trend is assumed to follow a random walk

$$\begin{bmatrix} \overline{y}_{hp,t} \\ \overline{y}_{hp,t-1} \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \overline{y}_{hp,t-1} \\ \overline{y}_{hp,t-2} \end{bmatrix} + \begin{bmatrix} \omega_{\gamma t} \\ 0 \end{bmatrix},$$

and y_t^c is assumed to be white noise. In addition, the noise to signal ratio is assumed to be fixed,

$$\frac{\sigma^2_{y_t^c}}{\sigma^2_{\omega\gamma}} = 1600.$$

Instead of imposing this restriction, we let the data speak and estimate this ratio using the information contained in the unemployment, inflation and investment rates. Some advantages of a model–based approach are that the filters implied by the model are consistent with each other and with the data. In addition, they automatically adapt to the ends of the sample and, if desired, root mean squared errors can be calculated.

2.2 Okun's Law

The negative correlation between the output gap and cyclical unemployment specified by Okun's law is expressed by means of the following equation

$$U_t = \phi_u U_{t-1} + (1 - \phi_u)\overline{U}_t + \phi_y(L)y_t^c + v_{ut},$$

where $\overline{U}_t$ is the trend component, v_{ut} is assumed to be an i.i.d. $N(0, \sigma_{uv}^2)$ sequence and $\phi_y(L)$ is a polynomial in the lag operator such that $\phi_y(1) < 0$. Since the output gap follows an $AR(2)$ process, our cyclical unemployment specification is, in principle, rather flexible. In contrast to the assumptions of Apel and Jansson (1999) and Camba-Méndez and Palenzuela (2003), we allow the output gap to affect the unemployment rate with some lags as suggested by some empirical evidence which shows that firms usually adjust employment slowly.

The non–accelerating inflation rate of unemployment or NAIRU, $\overline{U}_t$, is allowed to follow either an $I(2)$ or an $I(1)$ process. That is,

$$\overline{U}_t = \gamma_{ut} + \overline{U}_{t-1},$$

where

$$\gamma_{ut} = \rho_u \gamma_{ut-1} + \omega_{ut},$$

$0 \leq \rho_u \leq 1$, and ω_{ut} is assumed to be an i.i.d. $N(0, \sigma_{u\omega}^2)$ sequence. Thus, if $\rho_u = 1$, then $\triangle\overline{U}_t$ is $I(1)$. But if $\rho_u = 0$, then $\overline{U}_t$ is just a random walk. As pointed out by Laubach (2001), the assumption that the NAIRU is a random walk could be convenient for the US but not for other countries such, for example, as the European ones where this random walk process NAIRU is believed to contain a random drift term. We check this hypothesys later in the paper. Note that our specification is also general enough to allow the unemployment rate to fluctuate around a stationary NAIRU when $\omega_{ut} = 0$.

2.3 Investment

Among the most important regularities that the empirical research on business cycles has found, there is particularly one which can be used to obtain additional valuable information about the cyclical position of the economy. Namely, investment strongly comoves with output but with more volatility (Burnside, 1998, Harvey and Trimbur, 2003). This stylized fact implies that the deviation of the investment rate, $x_t \equiv I/GDP$, from its long-run trend, $\overline{x}_t$, is markedly procyclical. Therefore, a convenient flexible form to model the comovement of the investment rate with the output gap is given by the following equation,

$$x_t = \beta_x x_{t-1} + (1 - \beta_x)\overline{x}_t + \beta_y(L)y_t^c + v_{xt}, \tag{2}$$

where v_{xt} is assumed to be an i.i.d. $N(0, \sigma_{xv}^2)$ sequence and, given that the investment rate is procyclical, $\beta_y(L)$ is a polynomial in the lag operator such that $\beta_y(1) > 0$. Equation (2) implies that the trend is the long-run investment rate consistent with an output gap equal to zero. We interpret equation (2) as a reduced-form which stands for the correlation between the cyclical component of the investment rate and output, and not as a structural investment equation.

As in the case of the NAIRU, the trend component of the investment rate is allowed to follow either an $I(1)$ or an $I(2)$ model. That is,

$$\overline{x}_t = \gamma_{xt} + \overline{x}_{t-1},$$

where

$$\gamma_{xt} = \rho_x \gamma_{xt-1} + \omega_{xt},$$

$0 \leq \rho_x \leq 1$, and ω_{xt} is assumed to be an i.i.d. $N(0, \sigma^2_{\omega x})$ sequence.

2.4 The Phillips curve

Our Phillips curve specification relies on the p-bar model proposed by McCallum (1994), which is based on Mussa (1981). In the basic version of this model, the current price level, p_t, adjusts according to

$$p_t - p_{t-1} = E_{t-1}(\overline{p}_t - \overline{p}_{t-1}) + \eta_y y^c_t,$$

where $\overline{p}_t$ is the price level consistent with $y^c_t = 0$ and E_t denotes expectation conditional on the information up to time t. However, as shown by Fuhrer and Moore (1995) for the US, inflation exhibits a strong persistence. In the spirit of the p-bar model, we assume that inflation adjusts according to the following equation

$$\pi_t = (1 - \sum_{i \geq 1} \mu_{\pi i})\overline{\pi}_t + \mu_\pi(L)\pi_{t-1} + \eta_y y^c_t + \upsilon_{\pi t}, \tag{3}$$

where $\upsilon_{\pi t}$ is assumed to be an i.i.d. $N(0, \sigma^2_{\pi \upsilon})$ sequence, $\mu_\pi(L) = \sum_{i \geq 1} \mu_{\pi i} L^i$ is a polynomial in the lag operator and $\overline{\pi}_t$ is the long-run inflation rate or core inflation consistent with $y^c_t = 0$.

This specification of the Phillips curve presents several interesting properties. First, the term $\overline{\pi}_t$ is closely related to the traditional definition of core inflation. Thus, following Bryan and Cecchetti (1994), core inflation is the component of price changes that is expected to persist over a horizon of several years, $t + j$, that is

$$\overline{\pi}_t = E_t \pi_{t+j}. \tag{4}$$

Note however that $\overline{\pi}_t$ is defined in (4) as an estimator with no model behind it, whereas our $\overline{\pi}_t$ is defined as an unobserved component in equation (3). As in the New Phillips curve (Galí and Gertler, 1999, Galí, Gertler and López-Salido, 2001, Roberts, 2001b), we assume that forward-looking firms increase prices taking into account the expected inflation for a long forecast horizon, while traditional sources of inflation inertia justify the presence of different inflation lags in the Phillips curve. Restricting the coefficient of $\overline{\pi}_t$ in equation

(3) implies that core inflation is the long-run level of inflation consistent with an output gap equal to zero:

$$\pi_t = \frac{(1 - \sum_{i \geq 1} \mu_{\pi i})}{1 - \mu_\pi(L)} \overline{\pi}_t + \frac{\eta_y}{1 - \mu_\pi(L)} y_t^c + \frac{1}{1 - \mu_\pi(L)} \upsilon_{\pi t}.$$

Second, our Phillips curve is flexible enough to encompass other specifications usually found in the literature as particular cases under some restrictions. For example, backward-looking specifications as the one estimated by Rudebusch and Svensson (1999) for the US assume that the sum of the coefficients of $\mu_\pi(L)$ is unity, so that there is no trend component $\overline{\pi}_t$ and inflation π_t enters equation (3) in first differences.

Finally, core inflation is modelled as an $I(1)$ or an $I(2)$ process

$$\overline{\pi}_t = \gamma_{\pi t} + \overline{\pi}_{t-1},$$

where

$$\gamma_{\pi t} = \rho_\pi \gamma_{\pi t-1} + \omega_{\overline{\pi} t},$$

$0 \leq \rho_\pi \leq 1$, and $\omega_{\overline{\pi} t}$ is assumed to be an i.i.d. $N(0, \sigma^2_{\omega \overline{\pi}})$ sequence.

2.5 Model Estimation

Our approach to estimate the unknown parameters in the model is to cast it into state-space form and use the Kalman filter for likelihood evaluation. Then, at a later stage, we use a smoothing algorithm to obtain estimates of the unobserved components together with their mean squared errors. We use quarterly data for the US economy from 1947:I to 2003:I. The data are described in Appendix A. There are five missing values, but that poses no problem for the Kalman filter.

Before writing the equations in state space form, we have to specify whether the trend components are $I(1)$ or $I(2)$ and the degrees of the polynomials $\phi_y(L)$, $\beta_y(L)$ and $\mu_\pi(L)$. After performing some unit root tests and inspecting the graphs of the four variables, we concluded that the four variables are at least $I(1)$. Based on economic theory, we specified at first degree two for the polynomial $\phi_y(L)$, degree one for the polynomial $\beta_y(L)$ and degree four for the polynomial $\mu_\pi(L)$.

Assuming the four variables to be $I(1)$, first we allowed the coefficients ρ_y, ρ_u, ρ_x and ρ_π to be positive and strictly less than one. After estimating the model, we obtained estimates of these coefficients which were very close to zero and statistically insignificant. Then, we eliminated from the model the less significant coefficient. Proceeding in this way, we got to the conclusion that all four coefficients should be eliminated, implying that the four variables in our model are $I(1)$ and not $I(2)$. In addition, we confirmed the specification of the degrees of the polynomials $\phi_y(L)$, $\beta_y(L)$ and $\mu_\pi(L)$, since the coefficients were significant.

Our model, with the previous specification, can be put into state-space form as follows. Define the following matrices

$$W = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & -\theta_1^2 & 2\theta_1 \cos\theta_2 \end{bmatrix},$$

$$\alpha_t = \begin{bmatrix} \overline{y}_t \\ \overline{U}_t \\ \overline{x}_t \\ \overline{\pi}_t \\ y_{t-2}^c \\ y_{t-1}^c \\ y_t^c \end{bmatrix}, \quad H = \begin{bmatrix} \sigma_{\gamma\omega}^* & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{u\omega}^* & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{x\omega}^* & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{\pi\omega}^* & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{y\omega}^* \end{bmatrix},$$

$$Z = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1-\phi_u & 0 & 0 & \phi_2 & \phi_1 & \phi_0 \\ 0 & 0 & 1-\beta_x & 0 & 0 & \beta_{y_1} & \beta_{y0} \\ 0 & 0 & 0 & 1-\sum_{i=1}^4 \mu_{\pi_i} & 0 & 0 & \eta_y \end{bmatrix},$$

and

$$G = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{uv}^* & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{xv}^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix},$$

where $\sigma_{\gamma\omega}^* = \sigma_{\gamma\omega}/\sigma_{\pi v}$, $\sigma_{u\omega}^* = \sigma_{u\omega}/\sigma_{\pi v}$, $\sigma_{x\omega}^* = \sigma_{x\omega}/\sigma_{\pi v}$, $\sigma_{\pi\omega}^* = \sigma_{\pi\omega}/\sigma_{\pi v}$, $\sigma_{y\omega}^* = \sigma_{y\omega}/\sigma_{\pi v}$, $\sigma_{uv}^* = \sigma_{uv}/\sigma_{\pi v}$, and $\sigma_{xv}^* = \sigma_{xv}/\sigma_{\pi v}$. Then, α_t is the state vector, the parameter $\sigma_{\pi v}^2$ is concentrated out of the likelihood, and the state-space equations are

$$\begin{aligned} \alpha_{t+1} &= W\overline{\gamma}_y + T\alpha_t + H\epsilon_t \\ z_t &= Z\alpha_t + G\epsilon_t, \end{aligned}$$

where $z_t = [y_t, U_t - \phi_u U_{t-1}, x_t - \beta_x x_{t-1}, \pi_t - \sum_{i=1}^4 \mu_{\pi_i}\pi_{t-i}]'$ and $\mathrm{Var}(\epsilon_t) = \sigma_{\pi v}^2 I$. The parameter $\overline{\gamma}_y$ is also concentrated out of the likelihood. The filter starts filtering at $t = 5$, so that we condition on the first four observations of each series.

The previous state-space model is non-stationary and the initial conditions for the Kalman filter are not well defined. To overcome this difficulty, we use the approach of De Jong (1991). According to this approach, the initial state vector α_1 is modelled as partially diffuse and an augmented Kalman filter algorithm called the "diffuse Kalman filter" (DKF) is used to handle the diffuse part. As shown by De Jong and Chu-Chun-Lin (1994), the DKF can be collapsed to the ordinary Kalman filter after a few iterations.

The DKF can be used to evaluate the likelihood and thus the model parameters can be estimated by maximum likelihood.

After having estimated the model parameters, we can use a smoothing algorithm to obtain two-sided estimates of the unobserved components and their mean squared errors. We use the algorithm proposed by De Jong and Chu-Chun-Lin (2001). The diffuse part is $\delta = [\overline{y}_0, \overline{U}_0, \overline{x}_0, \overline{\pi}_0]'$, so that the initial state is $\alpha_1 = A\delta + W\overline{\gamma}_y + [0, x_1']'$, where $A = [I, 0]'$ and $x_1 = [y^c_{-1}, y^c_0, y^c_1]'$ has a known (stationary) distribution. Some technical details on the methodology and the optimization method are given in Appendix B.

3 Estimation Results

In Table 1 we present the estimates of the different model parameters, together with their $t-$statistics in parenthesis. It is seen that our estimation of the output gap is very significant in the Okun's law (ϕ_i), the Phillips curve (η_y) and the investment equation (β_{yi}). This suggests that the unemployment, inflation and investment rates contain very useful information about the cyclical position of the economy.

The results for the Okun's law indicate that there is a significant and sizable direct contemporaneous effect of business cycles on the unemployment rate but, as ϕ_u is significantly greater than zero, these effects last several quarters. Another noteworthy result is that the magnitude of σ_{v_u} is so small that Okun's law almost fits completely the unemployment rate. In the case of the investment rate, we obtain a similar picture. The contemporaneous correlation with the output gap is very significant, and there is also a substantial inertia in the investment rate since β_x is relatively high. Because the standard deviation of v_x is very small (close to 0.5 per cent), the decomposition between trend and cycle accounts almost entirely for the variation of the investment rate.

The last two columns of Table 1 present the estimation results of the Phillips curve. Again, the model performs extremely well in explaining the dynamics of inflation in the United States. The output gap is statistically significant suggesting that most of the business cycles fluctuations have been associated with a procyclical behaviour of inflation. Although the models are not directly comparable, the estimated value of η_y is higher than that estimated by Rudebusch and Svensson (1999). Compared with previous results in the literature, where the sum of the coefficients of past inflation ($\sum_{i\geq 1} \mu_i$) is usually around 1, we obtain lower values of this sum, which are equal to 0.4984, indicating that inflation inertia is rather important in the US economy. In other words, our estimates show an important role for core inflation, with a significant coefficient equal to 0.5016.

Figure 1 displays the output gap, the NAIRU, the core inflation and the trend of the investment rate for the US economy, as well as the estimated 90 per cent confidence intervals. In Figure 2 a), our estimation of the output gap is also compared with the cyclical component estimated with the HP filter. The correlation between both estimates of the output gap is relatively high (0.853), but we observe some important discrepancies. Thus, when we use the HP filter, the 1990-91 recession appears very mild compared to

Table 1
Kalman filter maximun likelihood estimates

Dependent Variables							
y_t		U_t		x_t		π_t	
θ_1	0.7586	ϕ_0	-0.2962	β_{y0}	0.6636	η_y	0.24098
	(18.75)		(-11.22)		(12.42)		(2.91)
θ_2	0.2972	ϕ_1	0.06233	β_{y1}	-0.6085	μ_1	0.5264
	(4.16)		(1.01)		(-11.03)		(6.09)
$\overline{\gamma}_y$	0.0083	ϕ_2	0.0943	β_x	0.7952	μ_2	-0.1038
	(24.61)		(1.93)		(11.25)		(-1.51)
		ϕ_u	0.7585			μ_3	0.3544
			(4.28)				(5.18)
						μ_4	-0.2786
							(-4.53)
σ_{wy}	0.0072	σ_{v_u}	0.0016	σ_{v_x}	0.0048	σ_{v_π}	0.01635
	(10.08)		(7.36)		(10.79)		(-)
$\sigma_{w\gamma}$	0.0049	σ_{w_u}	0.0017	σ_{w_x}	0.0027	σ_{w_π}	0.0057
	(9.11)		(3.13)		(2.75)		(3.02)

other post World War II episodes, whereas the growth in GDP during the second part of the nineties is compatible with a small output gap. However, our estimation of the output gap shows a more severe recession in 1990-91 and also an important cyclical expansion after these years, reaching a maximum at the beginning of 2000 that is similar to the one observed during the latest eighties. Figure 1 is also very illustrative about the performance of the NAIRU, which has remained quite stable from mid nineties onwards, around 5 per cent. This level is similar to the one observed in the fifties and sixties. Additionally, the confidence intervals indicate that expansions and recessions are precisely identified and, therefore, the difference between the current unemployment rate and the NAIRU is very useful for the conduct of economic policy. Our reading of these results is that they cast some doubts on recent criticisms (Staigner, Stock and Watson, 2001) about the statistical uncertainty in the estimation of the NAIRU and its usefulness for policy makers. As regards the performance of the core inflation rate, Figure 1 shows that desinflationary policies were very aggressive in the first half of the eighties, as Ball (1997) has pointed out, with a significant reduction of core inflation. In more recent years, core inflation has remained relatively stable.

As mentioned earlier, the results in Table 1 are obtained under the assumption that the rate of growth of real GDP is stationary around a constant ($\rho_y = 0$), but we have also estimated output gaps assuming that $\rho_y = 1$. In Figure 2 b), we show the estimates of the output gap under the two previous assumptions about the value of ρ_y. With the

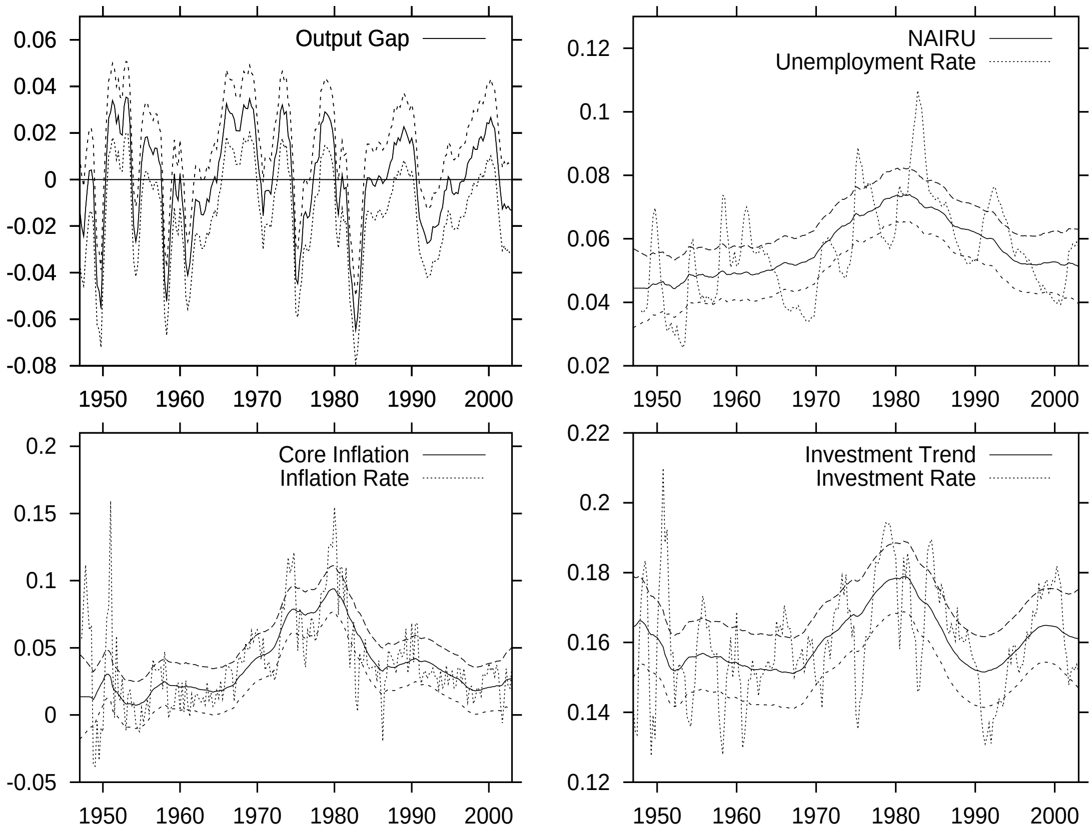

Figure 1: Output gap, NAIRU, Core inflation, and Investment rate trend. United States, 1948:I-2003:I.

exception of the second half of the sixties, both estimates of the output gap are almost identical, in particular during the last part of the sample, confirming again that the HP filter underestimates the output gap in the second half of the nineties also when $\rho_y = 1$ in our model. Note that this last assumption makes output an $I(2)$ variable in our model, like in the HP filter. The performance of inflation in this period helps explain the difference between our estimation of the output gap and that obtained with the HP filter. As we can see in Figure 1, the inflation rate increased significantly. Given our model equations, the rise of inflation is a by-product of a positive output gap.

4 Revisions and inflation forecasts

In this section, we perform two standard additional exercises aimed at analyzing some properties of our decomposition in comparison to alternative methods. In particular, we are interested in how important the revisions of our estimates are after new information becomes available and how useful the output gap and the core inflation are for predicting future inflation.

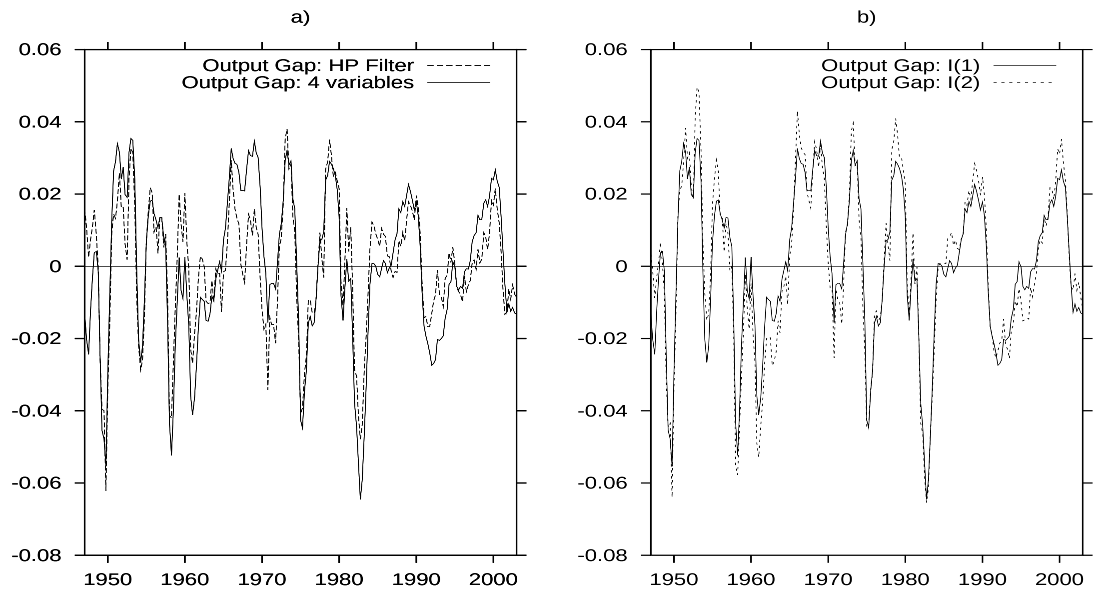

Figure 2: a) Output gap estimated with the HP filter and the four variables model, and b) Output gap estimated under the assumptions that the rate of growth of real GDP is stationary and that it is a random walk. United States, 1948:I-2003:I.

Several authors (Rünstler, 2002, Orphanides and van Norden, 2003, Camba-Mendez and Rodriguez-Palenzuela, 2003) have proposed comparing alternative models using their revision properties. That is, analyzing to what extent the availability of new information introduces changes into the previously estimated unobserved components. Let us define $z_{t/t+j}$ as the estimator of the unobserved component z_t based on all available observations up to time $t + j$. Thus, if $j = 0$ then $z_{t/t}$ is the real–time or concurrent estimate of z_t. As new data become available ($j = 1, 2, ...$), the model yields newer estimates of $z_{t/t+j}$ and, therefore, the difference between $z_{t/t}$ and $z_{t/t+j}$ is a measure of the revision made. As Orphanides and van Norden (2003) have shown, most of the revisions are due to the unreliability of end-of-sample estimates of the output gap. For this reason, we are interested in revisions due to the use of the information contained in the full sample, that is, in $z_{t/t} - z_{t/T}$, where T is the last observation in the sample. This revision is the quasi-final time estimate used by Orphanides and van Norden (2003), which is simply the rolling estimate based on the final data series but holding constant the set of estimated parameters for the whole sample. We compute the standard deviation of the revisions of trend components, comparing the results of our model with the following alternatives. We use the HP filter and the filter proposed by Baxter and King (1995) (henceforth BK filter) as well known examples of univariate methods. Additionally, we use two alternative multivariate models. The first one is our model with only two variables, inflation and output. This model is similar to the model proposed by Kuttner (1994) and will be referred to as Kuttner's model in the sequel. The second one is based on our model but

Table 2
Revisions 1960:I-1994:IV

	HP ($\lambda = 1600$)	BK (4,32)	Our model 4 vab.	Our model 3 vab.	Kuttner 2 vab.
Std. dev. of revisions in trend components					
Output	0.01680	0.01122	0.00670	0.00712	0.01215
Inflation	0.01362	0.01072	0.00809	0.00773	0.01024
Unemployment	0.00745	0.00477	0.00417	0.00522	–
Investment	0.00731	0.00562	0.00513	–	–
Correlation between concurrent and full sample estimates					
Output gap	0.52612	0.71761	0.95689	0.94558	0.76902
Δ Output gap	0.89958	0.78482	0.98131	0.97798	0.90093

excluding the investment equation.

In Table 2 we present the standard deviation of revisions of the trend components over the period 1960:I to 1994:IV, therefore excluding more than eight years at the beginning and at the end of the sample. This sample size allows us to use the HP and BK filters without ARIMA extrapolations at both ends of the sample. The results are very illustrative. The revisions for the HP filter are larger than for any other alternative method in each of the four variables. On the contrary, our preferred unobserved components model with four variables produces generally the smallest revisions. The standard deviation of revisions for core inflation is lower in the model with three variables than in the specification which includes investment, but at the cost of a higher standard deviation of revisions for the output gap and the unemployment rate trend component. For the BK filter, the revisions of the output gap are half way between those obtained with the HP filter and our model, and close to the ones computed with Kuttner's model. The same picture emerges again when we use the root mean squared (RMS) revision. Therefore, these results show that the estimates of the trend components are more stable when new observations become available in our preferred unobserved components model than in the alternative ones.

Table 2 also shows the correlations between concurrent and full sample estimates of the output gap for the five alternative models considered. The best results correspond to our unobserved components model with four variables, which exhibits a very high correlation. It is to be noted that the three unobserved components model yields a higher correlation than the HP and the BK filters. Finally, the last row in Table 2 shows the correlation between the change in the concurrent and full sample estimates of the output gap, since Walsh (2003) has pointed out that monetary policies which focus on the change of the output gap (speed limit policies) stabilize inflation and economic activity better than policies that focus directly on the output gap level. Again, our unobserved components model with four variables yields the highest correlation, making the results of our model very attractive as an input of stabilization policies. Given these results, from now on we

Table 3
Revisions 1985:1-1994:4

	HP	BK	Our model
	($\lambda = 1600$)	(4,32)	4 vab.
Std. dev. of revision			
Output	0.01252	0.00652	0.00313
Inflation	0.00997	0.00634	0.00413
Unemployment	0.00722	0.00363	0.00192
Investment	0.00690	0.00469	0.00563
Correlation between concurrent and full sample estimates			
Output gap	0.42880	0.74241	0.98948
Δ Output gap	0.85071	0.77303	0.97667

will only consider our model with four variables in future comparisons with the HP and BK filters.

In the previous exercise, the revisions of the unobserved components model have been computed using the parameters estimated with the whole sample. Another possibility is to estimate the parameters using a smaller sample, and then compute the revisions for the rest of the sample. We have estimated our model with observations from 1948:I to 1984:IV, and then we have repeated the preceding exercise for the period 1985:I to 1994:IV, which contains a complete cycle according to the three alternative decompositions. Thus, in this exercise the revisions are computed for quarters that have not been previously used in the estimation of the parameters of our model. As we can see in Table 3, the results show again that the smallest revisions of the output gap are obtained with our model. The same conclusion is reached when RMS revisions are used instead of standard deviations.

Leading indicator regressions (Stock and Watson, 1999) have often been used in the literature to assess the predictive power of some indicators over future changes in inflation. This motivates our final exercise, in which we analyze the performance as a leading indicator of the output gap (Rünstler, 2002, Orphanides and van Norden, 2003, Camba-Mendez and Rodriguez-Palenzuela, 2003), estimated with the models of the previous exercise, against the alternative of a benchmark autorregressive process for inflation given by

$$\pi_{t+j} - \pi_t = \mu(L)\nabla\pi_t + \xi_{t+j} \tag{5}$$

where $\nabla\pi_t = \pi_t - \pi_{t-1}$ and $\mu(L)$ is a polynomial in the lag operator such that its order is determined using the Schwarz information criterion. Specifically, we first estimate the following equation for the different measures of the output gap and, following Cogley (2002), deviations from core inflation

$$\pi_{t+j} - \pi_t = \beta(L)\left(\pi_t - \overline{\pi}_{t/t}\right) + \eta_y \widehat{y}^c_{t/t} + \varepsilon_{t+j},$$

where the order of $\beta(L)$ is equal to that of $\mu(L)$ in (5). It is important to notice that in this equation we are using only concurrent estimates of the output gap and core inflation, that

Table 4
Relative RMSE of inflation forecasts

	Rolling regressions			1985:I-1994:IV		
	HP	BK	Our model	HP	BK	Our model
$j = 2$	0.91570	0.85860	0.82925	1.00061	0.89884	0.90238
$j = 4$	0.91097	0.82373	0.79950	1.08744	0.96421	0.89643
$j = 6$	0.85965	0.77048	0.72983	0.91794	0.78820	0.61735
$j = 8$	0.84921	0.69673	0.67866	0.87277	0.65853	0.48532
$j = 12$	0.77778	0.66771	0.63335	0.95257	0.70594	0.48839

is, we are not including the information about the inflation rate contained in the whole sample. We then compute the root mean squared error (RMSE) of the inflation forecasts (the estimated ε_{t+j}) relative to the RMSE of the benchmark forecasts (the estimated ξ_{t+j}).

Table 4 shows the averages of the relative RMSE of inflation forecasts over the periods i to 1999:IV, for i running from 1970:I to 1990:I. The unobserved components model yields the lowest average relative RMSE of inflation forecasts for the different time horizons. This result is relatively stable for the different periods considered in the rolling regressions, and it is also reproduced in the case of a moving window of forty quarters starting in 1970:I. Finally, in order to avoid the criticism that the model parameters have been estimated using information from all the periods in which the RMSE of inflation forecasts are computed, the last three columns in Table 4 show the results when our model is estimated with data from 1948:I to 1984:I, as in the previous revision exercise, and the forecasting ability of the three procedures is evaluated over the period 1985:I to 1994:IV. Again, the unobserved components model produces better inflation forecasts than the HP and BK filters. Overall, these results indicate that the output gap and core inflation obtained with our multivariate model contain useful information for forecasting the inflation rate.

5 Conclusions

In this paper we have proposed an unobserved components model that provides estimates of the NAIRU, core inflation and the output gap for the United States. The model exploits the rich information about the business cycle simultaneously contained in the GDP and the unemployment, inflation and investment rates, to decompose these four variables into trend and cyclical movements. The unknown parameters in the model have been estimated by maximum likelihood using a Kalman filter initialized with a partially diffuse prior, and the unobserved components have been estimated using a smoothing algorithm. Although the correlation between the output gap estimated with this method and that obtained with the HP filter is relatively high, there are some important discrepancies, particularly in the second half of the nineties. Contrary to the HP filter, our method also works well at the end of the sample and, thus, it is very appropriate to infer how current economic conditions affect output, inflation and the unemployment rate.

Our results also show that the output gap estimated with our model is a very significant variable in Okun's law, the Phillips curve and the investment rate equation. These results confirm that the dependent variables in these equations improve the precision of the GDP decomposition into its trend and output gap components. Finally, we have verified that the revisions of output gap estimates when new information becomes available are lower with our preferred model than with the HP and the BK filters. In addition, our model also performs better than these last filters in forecasting inflation at different time horizons. The results obtained in this paper illustrate the usefulness of our decomposition for the conduct of stabilization economic policies in real time.

APPENDIX A: DATA SOURCES

The data set is available at **http://iei.uv.es/˜rdomenec/output/output.htm**. The variables contained in this file are the following:

- *Quarters:* from 1946:I to 2003:I.
- $\ln GDP$: log of Real Gross Domestic Product, Billions of Chained 1996 Dollars, SAAR. Source: BEA, Table 1.10, Line 1.
- π: quarterly inflation rate, defined as $4(\ln P_t - \ln P_{t-1})$, where P_t is the geometric average of the monthly price levels. Source: BLS, CPI Urban Consumer, all items, 1982-84=100, SA.
- U : unemployment rate, defined as the average of the monthly unemployment rates. Source: BLS, household survey, SA.
- x: nominal investment rate, defined as I/Y, where I is the nominal gross private domestic investment (Bil. $, SAAR), source: BEA, Table 5.4, Line 1, and Y is the nominal gross domestic product, (Bil. $, SAAR), source: BEA, Table 1.9 Line 1.

APPENDIX B: TECHNICAL DETAILS

In order to implement the methodology used in this paper, we have used a set of MATLAB programs written by V. Gómez. The code and the data sets are available at the Internet address: **http://iei.uv.es/˜rdomenec/output/output.htm**.

The estimation of the model parameters is performed by maximizing the so called "diffuse likelihood" (De Jong, 1991). It can be shown that maximizing the concentrated log–likelihood is equivalent to minimizing a nonlinear sum of squares function. To this end, a routine has been written in MATLAB that implements the Levenverg–Marquardt method, although the standard MATLAB routine "**lsqnonlin**" of the OPTIMIZATION TOOLBOX can also be used.

Letting θ be the vector of parameters to be estimated, the nonlinear sum of squares function that is minimized can be written as $F(\theta) = e'(\theta)e(\theta)$, where $e(\theta)$ is a vector of residuals that can be computed by the diffuse Kalman filter. In terms of $F(\theta)$, the log–likelihood $L(\theta)$ can be expressed as

$$L(\theta) = \text{const.} - \frac{n}{2} \ln F(\theta),$$

where n is the total number of observations. Under the usual assumptions, the estimator $\hat{\theta}$ of θ is asymptotically distributed as $N(\theta, I^{-1}/n)$, where I is the information matrix. This last matrix can be estimated by

$$\begin{aligned}\hat{I} &= -\frac{1}{n}\frac{\partial^2 L(\theta)}{\partial\theta\partial\theta'}_{|\theta=\hat{\theta}} \\ &= \frac{1}{2}\frac{\partial^2 \ln F(\theta)}{\partial\theta\partial\theta'}_{|\theta=\hat{\theta}},\end{aligned}$$

where the derivatives can be computed numerically. This is the method that has been implemented in MATLAB and used to compute standard errors for the model parameters.

The smoothing algorithm provides the standard errors of the unobserved components, with which the confidence intervals can be calculated. Other sources of uncertainty, like that due to parameter estimation could be taken into account through simulation, but this has not been implemented for this paper.

References

[1] Apel, M. and Jansson, P. (1999), "A Theory-Consistent System Approach for Estimating Potential Output and the NAIRU". *Economics Letters*, 64, 271-75.

[2] Ball, L. (1997), "Desinflation and the NAIRU". *NBER Macroeconomic Annual,* 167-92.

[3] Baxter, M. and King, R. G. (1995), "Measuring Business Cycles: Approximate Band–pass Filters for Economic Time Series", NBER Working Paper 5022.

[4] Bryan, M. F. and Cecchetti, S. G. (1994), "Measuring Core Inflation", in N. G. Mankiw, ed., *Monetary Policy.* University of Chicago Press.

[5] Burnside, C. (1998), "Detrending and Business Cycle Facts: A Comment". *Journal of Monetary Economics*, 41, 513-532.

[6] Camba-Méndez, G. and Palenzuela, D. R. (2003), "Assessment Criteria for Output Gap Estimates". *Economic Modelling*, 20, 529-62.

[7] Canova, F. (1998), "Detrending and Business Cycle Facts". *Journal of Monetary Economics*, 41, 475-512.

[8] Clarida, R., Galí, J., and Gertler, M. (1999), "The Science of Monetary Policy: A New Keynesian Perspective". *Journal of Economic Literature*, 37, 1661-1707.

[9] Cogley, T. (2002), "A Simple Adaptative Measure of Core Inflation". *Journal on Money, Credit and Banking*, 34(1), 94-113.

[10] De Jong, P., (1991), "The Diffuse Kalman Filter". *Annals of Statistics*, 19, 1073-1083.

[11] De Jong, P. and Chu-Chun-Lin, S. (1994), "Fast Likelihood Evaluation and Prediction for Nonstationary State Space Models", *Biometrika*, 81, 133-142.

[12] De Jong, P. and Chu-Chun-Lin, S. (2001), "Smoothing With an Unknown Initial Conditon". Mimeo.

[13] Furher, J.C. and Moore, G. R. (1995), "Inflation Persistence". *Quaterly Journal of Economics*, 109, 127-159.

[14] Galí, J. and Gertler, M. (1999), "Inflation Dynamics: A Structural Econometric Analysis". *Journal of Monetary Economics*, 44, 195-222.

[15] Galí, J., Gertler, M. and López-Salido, D. (2001), "European Inflation Dynamics". *European Economic Review*, forthcoming.

[16] Gerlach, S. and Smets, F. (1999), "Output Gaps and Monetary Policy in the EMU Area". *European Economic Review*, 43, 801-812.

[17] Gómez, V. (1999), "Three Equivalent Methods for Filtering Finite Nonstationary Time Series". *Journal of Business and Economic Statistics*, 17, 109-116.

[18] Gordon, R. J. (1997), "The Time-Varying NAIRU and its Implications for Economic Policy". *Journal of Economics Perspectives*, 11(1), 11-32.

[19] Harvey, A. C. and Trimbur, T. M. (2003), "General Model-Based Filters for Extracting Cycles and Trends in Economic Time Series". *The Review of Economis and Statistics,* 82(2).

[20] Hodrick, R. and Prescott, E. C. (1997), "Post-war US Business Cycles: An Empirical Investigation". *Journal of Money, Credit and Banking*, 29(1), 1-16.

[21] King, R. G. (2000), "The New IS-LM Model: Language, Logic, and Limits". Federal Reserve Bank of Richmond *Economic Quarterly*, 86(3), 45-103.

[22] Kuttner, K. N. (1994), "Estimating Potential Output as a Latent Variable". *Journal of Business and Economic Statistics* 12, 361-8.

[23] Laubach, T. (2001), "Measuring the NAIRU: Evidence from Seven Economies". *Review of Economics and Statistics*, 83(2), 218-231.

[24] Mankiw, N. G. (2001), "The Inexorable and Mysterious Tradeoff Between Inflation and Unemployment". *The Economic Journal*, 111 (May), C45-C61.

[25] McCallum, B. T. (1994), "A Semi-classical Model of Price Level Adjustment". *Carnegie-Rochester Conference Series on Public Policy*, 41, 251-284.

[26] Mc Morrow, K. and Roeger, W. (2001), "Potential Output: Measurement Methods, New Economy Influences and Scenarios for 2001-2010. A Comparison of the EU15 and the US". Economic Papers no. 150. Directorate-General for Economic and Financial Affairs. European Commission.

[27] Mussa, M. (1981), "Sticky Individual Prices and the Dynamics of the General Price Level". *Carnegie-Rochester Conference Series on Public Policy*, 15, 261-296.

[28] Orphanides, A. and van Norden, S. (2003), "The Unreliability of Output-gap Estimates". *The Review of Economics and Statistics*, 84(4), 569-83.

[29] Roberts, J. M. (2001a), "Estimates of the Productivity Trend Using Time-Varying Techniques". *Contributions to Macroeconomics*, 1(1), Article 3.

[30] —— (2001b), "How Well Does the New Keynesian Sticky-Price Model Fit the Data?". Board of Governors of the Federal Reserve System.

[31] Rudebusch, G. D. and Svensson, L. E. O. (1999), "Policy Rules for Inflation Targeting", in J. B. Taylor, ed., *Monetary Policy Rules*. University of Chicago Press.

[32] Rünstler, G. (2002), "The Information Content of Real-Time Output Gap Estimates: An Application to the Euro Area". ECB Working Paper no. 182.

[33] Stadler, G.W. (1994), "Real Business Cycles". *Journal of Economic Literature* (38), pp. 1750-83.

[34] Staigner, D., Stock, J. H., and Watson, M. W. (2001), "Prices, Wages and the US NAIRU in the 1990s". NBER Working Paper 8320.

[35] Stock, J. H., and Watson, M. W. (1998), "Median Unbiased Estimation of Coefficient Variance in a Time-Varying Parameter Model". *Journal of the American Statistical Association*, 93, 349-58.

[36] Walsh, C. E. (1998), *Monetary Theory and Policy*. The MIT Press.

[37] Walsh, C. E. (2003), "Speed Limit Policies: The Output Gap and Optimal Monetary Policy". *American Economic Review*, 93(1), 265-278.