R&D-EXPENDITURE IN AN ENDOGENOUS GROWTH MODEL

*María Jesús Freire-Serén**

D-2000 - 03

May 2000

* University of Vigo.

I am indebted to Jaime Alonso, Jordi Caballé, Angel de la Fuente, Lorenzo Serrano and two anonymous referees for their helpful discussions and comments. Of course, all errors that remain are entirely my own. This paper was written as part of a research project cofinanced by the Fundación Caixa Galicia. Financial support from the Spanish Ministry of Education (through DGICYT grants no. PB95-130 and SEC99-1189) is also gratefully acknowledged.

Abstract

The aim of this paper is to analyze the role that aggregate R&D-expenditure play in economic growth. We introduce a technology of innovation based on expenditure that generates endogenous sustainable growth in abscence of any scale effect. Thus, this R&D-model with an expanding variety of products permits us to study the effects of some fiscal policies. In particular, we analyze the impact that the subsidy to R&D-investment and the subsidy to physical capital accumulation have on the long-run growth rate. For the empirical cross-country analysis we directly derive a structural econometric model from this R&D-endogenous model.

JEL Classification: O30, O38, O40, O47

Keywords: R&D-Expenditure, Endogenous Growth, Fiscal Policy, Cross-Country Evidence, Technological Change.

1. Introduction.

In the last decade, the importance of the creation and diffusion of ideas to economic development has been well recognized. This fact is corroborated by the increasing participation of the governments subsidizing these R&D-activities. In order to justify the relevance of this public intervention, two issues arise. The first would be to analyze the impacts that a subsidy to this activity has on long-run economic growth. The second would consist in verifying, using cross-country data, whether the income that is devoted to financing innovation activities is translated into growth of per capita income.

In this sense, the aim of the present paper is to theoretically and empirically analyze the role that aggregate R&D-expenditures play in determining the growth of per capita income. We develop an R&D-based model of economic growth in the line of those proposed by Romer (1990), where the core of the economy consists of differentiated physical capital stock. The variety of this physical capital stock is expanded by R&D investment.

Generally, these R&D-based models of growth consider an R&D-technology that uses skilled labor as a unique input. However, we will use a technology of innovation based on expenditure instead of labor. Now then, by R&D-expenditure we mean the income devoted to finance the total cost of the R&D-activity. Although we can find other models where R&D is made with income (see, Rivera-Batiz and Romer, 1991; Barro and Sala-i-Martin, 1995; Aghion and Howitt, 1998), the specification that we propose is different. Following Jones (1995b), we adopt an R&D-technology that incorporates two kinds of externalities. First, the productivity of the R&D-expenditure positively depends on the level of ideas discovered in the economy. Second, the aggregate level of the R&D-expenditure drives the marginal productivity of the R&D-expenditure down at the firm level. This means that the higher the aggregate R&D-expenditure is, the larger the number of firms developing the innovation activity and, because of duplication, the smaller the individual probability of discovering new ideas. This specification avoids the problem of the scale effects common in the R&D-based models of economic growth.

The early models of Schumpeterian growth (e.g., Romer, 1990; Grossman and Helpman, 1991; Aghion and Howitt, 1992) exhibited scale effects predicting that the growth rate of the economy is proportional to the effort that the economy allocates to research. However the empirical evidence is incompatible with this scale effect (see Jones, 1995b; Dinopoulos and Thompson, 1999). Several recent papers on scale and growth (Jones, 1995a; Kortum, 1997; Segerstrom, 1998) follow different strategies to present a model in which long-run per capita income is proportional to the rate of population growth. Thus, the scale effect shows up in the level of per capita income instead of its growth rate. In these papers, fiscal policies may affect the level of income, but not its long-run growth rate --in Segerstrom (1998), for instance, the government could implement other public policies, like setting novelty requirements in the patents, to promote long-run economic growth. On the other hand, Dinopoulos and Thompson (1998), Young (1998), and Peretto (1998) describe an economy that also avoids the scale effects in the growth rate. They suppose that firms do two types of R&D, improve existing products and develop new varieties. In this way, they assume that R&D becomes progressively more difficult over time. With respect to the fiscal policy, in Young (1998) an R&D-subsidy neither affects this long-run growth rate, although proportional subsidies to R&D can have effects on the level of consumer utility. However Dinopoulos and Thompson (1998) find that a subsidy to quality-enhancing R&D raises the long-run rate of quality growth.

The main feature of the paper presented here is that the absence of any scale effect in a model with an expanding variety of products is also combined with a long-run growth rate that does not depend linearly on the population growth rate. R&D technology is based on expenditure (i.e., foregone output), a private input that consumers choose and can be increased when income rises. Therefore, sustainable growth is due to this feedback relation between R&D and income, and the long-run growth rate is fully endogenous. The most interesting contribution of the paper is that some fiscal policies will now have effects on economic growth. In particular, we will show that a subsidy to R&D-expenditure and a subsidy to physical capital investment both increase the long-run growth rate. Moreover, we will numerically compute the combination of both subsidies that maximizes this long-run growth rate.

The paper also empirically analyzes how R&D-expenditure affects productivity growth using cross-country data. There have been many studies that have estimated the impact of R&D-investment on productivity at the firm and industry level. However, there are not enough articles that contrast the evidence of those endogenous models by using international aggregate data. These previous cross-country studies are based either on "ad-hoc" equations about the total factor productivity (see Coe and Helpman, 1995; Coe et al. 1997, Engelbrecht, 1997) or on convergence equations approximately similar to the one developed by Mankiw, Romer and Weil (1992), (see Nonneman and Vandhoudt, 1996; Murthy and Chien, 1997; de la Fuente, 1997). In the present paper we estimate an equation directly obtained from the optimizing behavior of consumers and the profit maximization of firms. Thus, one contribution of our analysis is that we directly derive a structural econometric model to test empirically an R&D-model of endogenous growth.

The paper is organized as follows. In Section 2 we present the model. The competitive equilibrium and the balanced growth path are analyzed in Section 3. Section 4 presents the growth effects of fiscal policy. Section 5 studies the optimal solution. In Section 6 we build an econometric model and show the empirical results. We present some conclusions in Section 7.

2. The Model.

We consider an R&D-based growth model with an infinite horizon and continuous time. The economy consists of five types of economic agents that we next introduce.

2.1 Final good sector.

This sector produces competitively a homogenous good by means of a Cobb-Douglas production function represented by:

$$Y_t = H_t^{\beta} L_t^{1-\alpha-\beta} \int_0^{N_t} X_{it}^{\alpha} di, \tag{1}$$

where Y_t is the final output in period t, and the production factors are, human capital, H_t, labor force, L_t, and the total variety of intermediate goods, X_{it}. We will assume that the stock of human capital and the labor force grow at exogenous and constant rates.

Since this is a competitive sector we can obtain the conditional demand functions for each factor by solving the standard maximization problem.

2.2 R&D sector.

This sector consists of a large number of equal and competitive firms. They take the savings of consumers to finance their research projects. With these projects, each of these firms attempts to discover new ideas, which will adopt the form of designs for new capital goods. The variation in the number of designs achieved by each firm is given by:

$$\dot{N}_{it} = \xi_t R_{it}, \tag{2}$$

where N_{it} denotes the stock of ideas achieved by firm i, R_{it} represents the investment or research expenditure of firm i, and ξ_t is the productivity factor of the R&D-activity. This productivity factor measures how many designs the firm can obtain for one unit of R&D-expenditure. Unlike the preceding growth models where R&D-technology is based on foregone consumption, we assume that this productivity factor is endogenously determined.[1] In particular, this factor embodies two external effects arising from the aggregate R&D-expenditure and from the aggregate stock of ideas, respectively. On one hand, the aggregate R&D-expenditure reduces the number of designs obtained for each unit of expenditure because this aggregate expenditure raises the competition for discovering new. This negative external effect of the aggregate R&D–expenditure derives from the redundancy in research. On the other hand, the stock of ideas increases the productivity of the R&D-activity through the knowledge spillovers. Hence, the productivity factor ξ_t can be parameterized as $\xi_t = \gamma R_t^{\lambda-1} N_t^{\phi}$, where λ and ϕ belong to $(0,1)$. Therefore, the variation on the aggregate number of designs is then given by:

$$\dot{N} = \gamma R_t^{\lambda} N_t^{\phi}. \tag{3}$$

From now on, we will assume that this R&D-process exhibits constant returns to scale at the aggregate level, that is, $\lambda + \phi = 1$.

Note that the stock of ideas or designs is now determined by intentional R&D-expenditure made by the households. After having invested R_t units of income, they are the owners of the new designs that this investment produces. Thus, the return of this investment will be the flows that the designs will yield. Each design is sold at a price P_{Dt} to a capital-goods producer. Therefore, the free-entry condition for this R&D sector can be stated as:

$$R_t(1 - s_R) = P_{Dt}\,\dot{N}, \tag{4}$$

where s_R denotes the rate at which the government subsidizes investment in research.

---

[1] Assuming that research uses foregone output as a input is equivalent to assuming that designs are produced in a separate sector that has the same technology as the final output sector. In this sense, our R&D-technology is similar to the one introduced by Rivera-Batiz and Romer (1991). However, unlike these authors, we assume that the productive factor of this technology is endogenously determined.

2.3 Intermediate goods sector.

At the moment of time t, this sector is composed of a number of firms on the interval $[0, N_t]$. These firms have purchased a design from the R&D-sector to produce the intermediate good i. They rent part of consumers' savings at a rate r_t to finance this production. We will assume the simplest technology given by $X_{it} = S_{it}$, where S_{it} represents consumers' savings.

Since firm i is the only seller of capital good i, the monopoly price of each intermediate good is determined by the elasticity of its demand given by $1/(1-\alpha)$. More precisely, this price is $p_{it} = r_t/\alpha$. By symmetry, each firm sets the same price and sells the same quantity of its durable goods, hence the monopolist's net profits are given by:

$$\Pi_{it} = p_{it}X_{it} - r_{it}S_{it}(1-s_k) = X_t(r_t/\alpha - r_t(1-s_k)) = r_tX_t\left(\frac{1-\alpha(1-s_k)}{\alpha}\right) = \Pi_t. \tag{5}$$

Note that the government also subsidizes the physical capital accumulation by reducing the production cost of the intermediate goods in the rate s_k.

The decision to produce a new capital good depends on a comparison between the price that the firms must pay for the use of the design, P_{Dt}, and the discounted stream of net revenue that this production will generate. Since the market for designs is competitive, their price set up as:

$$P_{Dt} = \int_t^{\infty} e^{-\int_t^{\tau} r_s ds}\Pi_\tau d\tau. \tag{6}$$

Finally, note that the symmetry allows us to state that the total physical capital stock of economy is given by $K_t = N_tX_t$.

2.4 Households.

The representative household's preferences are represented by

$$U=\int_0^{\infty}\frac{C_t^{1-\sigma}-1}{1-\sigma}e^{-\rho t}dt, \tag{7}$$

where C is consumption, ρ is the constant subjective rate of time preferences, and σ denotes the inverse of the constant elasticity of intertemporal substitution. In each period, this representative household is subjected to the flow of budget constraint

$$\dot{B}=(r_tB_t+w_t)(1-\tau)-C_t+T_t, \tag{8}$$

where B_t represents its stock of wealth, w_t denotes the total labor income, τ is a time-invariant income tax and T_t is a lump-sum tax. The households distribute their assets between both types of investment. Hence, the household's wealth is equal to the sum of the physical capital stock and the value of the designs she posses, i.e., $B_t=K_t+P_{Dt}N_t$.

By standard procedure, we get the optimal consumption path as

$$\frac{\dot{C}}{C}=\frac{1}{\sigma}((1-\tau)r_t-\rho). \tag{9}$$

Moreover, the household selects her portfolio between both types of investments by using an equilibrium non-arbitrage condition that equalizes their after-subsidy return.

2.5 Government.

The government faces a balanced budget constraint at each moment in time, i.e.,

$$T_t=\tau(r_tB_t+w_t)-s_kr_tS_t-s_RR_t. \tag{10}$$

It is assumed that the income tax parameter and both subsidy rates are kept constant over time, and only marginal variations in discrete moments in time can be introduced. Hence, the lump-sum tax T_t is the adjusting parameter of the government's budget constraint.

3. The Competitive Equilibrium.

In this section, we define the competitive equilibrium and describe its properties. To keep the dynamic analysis simple and highlight the effects of interest, we will suppose, as Romer (1990), that both human capital and the labor force are constant and equal to one.[2]

First, we must impose the market-clearing conditions. The savings allocated to the intermediate sector are transformed into an increase of the physical capital stock. We assume that capital is putty-putty so that $\dot{K} = S$.[3] Thus, the law of motion for capital stock is given by

$$\dot{K} = Y_t - C_t - R_t. \tag{11}$$

Moreover, by noting that $K_t = N_t X_t$, the output of the final good sector can be written as

$$Y_t = K_t^{\alpha} N_t^{1-\alpha}. \tag{12}$$

At this point, by noting that the monopoly price of the intermediate goods is equal to the marginal productivity of these goods in the final good sector, and given that $p_t = r_t/\alpha$, one can express the rate of return to the investment in the intermediate sector as follows:

$$r_t = \alpha^2 K_t^{\alpha-1} N_t^{1-\alpha}. \tag{13}$$

Finally, differentiating equation (6) with respect to time, and substituting the monopoly profits, we get that the dynamic evolution of the patentee's price is given by:

$$\dot{P}_D = P_{Dt} r_t - r_t X_t \left(\frac{1-\alpha(1-s_k)}{\alpha} \right). \tag{14}$$

---

[2] We build the model assuming that these two variables grew over time in order to derive a econometric model in accordance with data that we will use in the cross-country analysis.

[3] Note that depreciation does not exist here. If we consider a positive rate of depreciation, savings should be used not only to produce new types of capital goods, but also to replace the depreciation. However, the results of this paper would still remain valid.

This equation represents the non-arbitrage that relates the returns to both types of investments, and so it determines the equilibrium household's portfolio section.

Conditions (3)-(6), and (9)-(14) form a system of equations that can be solved for the competitive equilibrium given the initial endowments of designs and physical capital. The economy would be in a balanced growth path (BGP, henceforth) equilibrium when N, K and C grow at a constant rate, and P_D is constant over time. Thus, to characterize the equilibrium dynamics we define $X = K/N$, $Z = C/K$ and we get, from the competitive equilibrium equations, the reduced dynamic system in X, Z and P_D:

$$\dot{X} = X^{\alpha} - ZX - \left(\gamma P_D/(1-s_R)\right)^{(1/1-\lambda)} - \gamma^{1/1-\lambda}\left(P_D/(1-s_R)\right)^{(\lambda/1-\lambda)} X, \tag{15a}$$

$$\dot{Z} = \frac{Z}{\sigma}\left(\alpha^2 X^{(\alpha-1)}(1-\tau) - \rho\right) - ZX^{(\alpha-1)} + Z^2 + \left(\gamma P_D/(1-s_R)\right)^{1/1-\lambda} ZX^{-1}, \tag{15b}$$

$$\dot{P}_D = \alpha^2 X^{(\alpha-1)} P_D - \alpha\left(1-\alpha(1-s_k)\right)X^{\alpha}. \tag{15c}$$

Since the initial value of X is fully determined by the initial endowments of N and K, given these endowments, the dynamic system (15) form a complete characterization of the competitive equilibrium paths.

The stationary values of X, Z and P_D, which will be respectively denoted by $\overline{X}$, $\overline{Z}$ and $\overline{P}_D$, fully defines the BGP equilibria. The following result states the existence, uniqueness and stability properties of the BGP equilibria.

PROPOSITION 1. *(i) Consider the economy described by conditions (3)-(6) and (9)-(14). A unique, interior and locally saddle-path stable BGP equilibrium exists if this condition holds:*

$$\overline{P}_D < \left(\frac{(1-s_R)^{\lambda}}{\gamma} \left((1-\tau)\alpha^{1+\alpha}\left(1-\alpha(1-s_k)\right)^{(1-\alpha)}\right)^{1-\lambda} \right)^{1/1-\alpha(1-\lambda)}. \tag{16}$$

Otherwise, no interior BGP exists.

Since the interior BGP equilibrium satisfies the transversality condition, this Proposition also ensures local uniqueness and local convergence of the competitive equilibrium.[4]

In our model, unlike Jones (1995b), there is endogenously determined sustainable growth, even if there is no population growth. The key of this result is the assumption of a R&D-technology based on expenditure. The growth rate in the steady-state equilibrium is given by the expression:

$$\bar{g} = \gamma \left(\frac{\gamma \bar{P}_D}{(1 - s_R)} \right)^{\lambda/1-\lambda} . \tag{17}$$

As in Jones (1995b), this model does not exhibit scale effects, because the growth rate does not depend on the R&D-expenditure level.

4. The Fiscal Policy.

The introduction of this technology of innovation based on R&D-expenditure instead of labor generates endogenous growth without scale effects. For this reason, fiscal policy has a chance to generate growth effects. Hence, research agenda should deserve time for looking for those fiscal instruments that encourage growth. In this section we study the long-run growth effects of the fiscal policies considered in the paper, i.e., the subsidies to innovation and to the production of capital goods and the income tax. For that purpose, we suppose that the economy is initially on the balanced growth path in presence of all fiscal policies, and suddenly the government decides to implement an unanticipated, permanent, marginal increase in one of the fiscal parameters. We will analyze the impacts on the BGP equilibrium by using comparative static arguments.

---

[4] Proof of this Proposition is available from the author upon request.

PROPOSITION 2. *Consider the economy described in Proposition 1. Then*

(i) The long run effects of a marginal change in the subsidy to physical capital investment are:

$$\frac{d\overline{X}}{ds_K} < 0, \quad \frac{d\overline{P}_D}{ds_K} > 0, \quad \frac{d\overline{g}}{ds_K} > 0.$$

(ii) The long run effects of a marginal change in the subsidy to R&D-activity are:

$$\frac{d\overline{X}}{ds_R} < 0, \quad \frac{d\overline{P}_D}{ds_R} < 0, \quad \frac{d\overline{g}}{ds_R} > 0.$$

(iii) The long run effects of a marginal change in the income tax are:

$$\frac{d\overline{X}}{d\tau} < 0, \quad \frac{d\overline{P}_D}{d\tau} < 0, \quad \frac{d\overline{g}}{d\tau} < 0.$$

Proof: See Appendix A.

Proposition 2 confirms that the fiscal policies considered in this paper affect the long-run growth rate. In particular, we observe that τ has a negative growth effect, whereas s_k and s_R are both growth increasing. Under the assumption of the model, the government can stimulate long run growth either through a subsidy to R&D-expenditure, or by means of a subsidy to physical capital accumulation. While the growth effects of τ and s_R are quite trivial, it seems necessary to add some comments about the effects of s_k. The subsidy to physical capital accumulation reduces the production cost of the intermediate capital goods. Hence, the net profits obtained by monopolistic firms producing each good increase. This encourages not only the production of existing capital goods, but also the demand for new designs. This excess of demand raises the patent's price, and so the R&D-effort. Furthermore, Proposition 2 also shows that this subsidy has a negative effect on the ratio from physical capital to number of designs. Hence, the increase in the production level of each intermediate good generates an increase in the physical capital stock that is smaller than the increase in the number of designs generated by the subsidy.[5]

Since the government has two alternative fiscal instruments to encourage economic growth, and given that it possess limited resources, a question immediately arises. What of these two instruments are more effective to stimulate growth? From Appendix A, we observe that the growth-effect of a marginal increase in s_R is always larger than the growth effect of a marginal increase in s_k. However, this comparison is not valid to answer our question because these subsidies are implemented on different bases. Hence, the requirements of public resources to finance the marginal increase are different for each subsidy.

We have computed a numerical exercise to provide a coherent answer to the previous question. In particular, we have conducted a positive analysis of the relationship between the subsidy structure and growth. Following Pecorino (1993), the subsidy structure refers to the mix of both subsidies that satisfy the government budget constraint. Given certain amount of lump-sum tax the government chooses the subsidy structure that maximizes the long-run growth rate. Therefore, the government faces the problem of setting s_R and s_k that maximize (17) subject to (10). Since in the BGP equilibrium the government spending grows at the same rate as the output, we can assume that the lump-sum tax is a constant fraction of output.

Following Jones (1995b), we consider $\alpha = 0.33$ and $\rho = 0.03$. Moreover, we have assumed the standard value $\sigma = 1.5$.[6] Since from actual data it is difficult to infer values for the R&D-technological parameters, λ and γ, we have performed the exercise for three pairs of these

[5] From the above results, we can also state the negative impact that the taxes on both types of investment have on economic growth.

[6] The conclusions obtained are robust to marginal changes in the value of these parameters.

parameters. As Jones (1995b), we have supposed that λ is 0.75, 0.5 or 0.25. Finally, the value of γ was calculated by imposing a long run growth rate of 2% in absence of fiscal policy. The lump-sum tax takes different values from 0.5 to 10 percent of output.

In the environment described here, the main conclusion obtained from the simulations is that the growth-maximizing subsidy structure is one where s_R is strictly positive and s_k is non-positive.[7] We get that a mix of positive subsidies does not maximize the growth rate in the long run. In particular, if we rule out the possibility of negative subsidies, the exercise shows that the growth-maximizing subsidy structure is given by $s_R>0$ and $s_k = 0$. Even more, if we allow the existence of negative subsidies, we get that for $\lambda \in (0.5, 0.75)$ and for low values of the lump-sum tax, s_k will be negative so as to finance more R&D-activities. In these scenarios, the negative effect that a negative rate of s_k has on economic growth is smaller than the positive effect generated by the R&D-subsidy financed through the lump-sum tax plus this negative subsidy to physical capital investment. However, when $\lambda = 0.25$, since the negative externality from the aggregate R&D-expenditure is high, the positive effect generated by the increase on s_R does not compensate for the implementation of a capital tax. Therefore, in the later case, the combination of $s_R>0$ and $s_k<0$ does not maximize the growth rate in the long run.

In order to see the intuition behind these results, we have also realized two additional simulations. First, we have developed the same exercise as before, but fixing the lump-sum tax to zero. We have found that the growth rate when the government does not participate at all is smaller than if it subsidies the R&D-activity with the revenue collected from a tax on the production of capital goods (except for the case $\lambda = 0.25$). The second exercise has been realized under the assumption that s_R is zero. We have checked that, as we expected from Proposition 2, the subsidy to physical capital accumulation drives the long run growth rate up. However, this positive growth effect is quantitatively very moderate.

Therefore, our numerical exercise confirms Romer's conjecture about the effectiveness of a direct R&D-subsidy instead of the subsidy to physical capital accumulation. "Although all research is embodied in capital goods, a subsidy to physical capital accumulation may be a poor substitute for direct subsidies that increase the incentive to undertake research".[8] Evidently, if the government does not implement the direct subsidy to R&D-expenditure, the subsidy to physical capital indirectly encourages economic growth.

5. Welfare Properties of the Competitive Equilibrium.

There are two reasons to expect that the socially optimal level of innovation may be greater or less than that achieved in the market equilibrium. First, there are positive and negative externalities associated with R&D as was explained in section 2.2. Moreover, the fact that patent is purchased by a sector that engages in monopoly pricing provokes an under-investment in R&D (see Romer, 1990). Therefore, to complete the analysis, we next compare the market outcome with the socially optimal degree of R&D-expenditure and its corresponding long-run growth rate.

The social planning problem of our economy consists in maximizing (7) subject to the constraints (3) and (11). This problem is a dynamic optimization problem with control variables C and R, and state variables K and N. Hence, by standard procedure, we find the first order

[7] The tables containing the simulation results are available from the author upon request.
[8] See Romer (1990), pp. S99

conditions and rearrange the expressions to summarize the socially optimal dynamics by the following reduced system:

$$\dot{X} = X^{\alpha} - ZX - (\gamma \lambda P_D)^{(1/1-\lambda)} - \gamma^{1/1-\lambda}(\lambda P_D)^{(\lambda/1-\lambda)} X, \tag{19a}$$

$$\dot{Z} = \frac{Z}{\sigma}\left(\alpha X^{(\alpha-1)} - \rho\right) - ZX^{(\alpha-1)} + Z^2 + (\gamma \lambda P_D)^{1/1-\lambda} ZX^{-1}, \tag{19b}$$

$$\dot{P}_D = \alpha X^{(\alpha-1)} P_D - (1-\alpha)X^{\alpha} - (1-\lambda)\lambda^{-1}(\gamma \lambda P_D)^{1/1-\lambda}. \tag{19c}$$

By comparing the previous system with the dynamic system (15) obtained in Section 3, we can establish the difference between the equilibrium and the socially optimal outcome. In this section, we will concentrate on the BGP. In particular, we will numerically compare the R&D-expenditure and the growth rate along the BGP that emerge from the equilibrium in absence of fiscal policy with their socially optimal counterparts. Since in both problems we cannot get analytical solutions for the stationary variables of X, Z and P_D, we will make a numerical comparison. We will use the same parameter values as in the simulations of the previous section.

Within the limits of the model, we obtain that the market outcome is characterized by an under-investment in R&D. More precisely, we find that the socially optimal growth rate and the socially optimal R&D expenditure are both higher for the three values of λ considered. In particular, while the competitive growth rate is 0.02 for any value of λ, the socially optimal rate of growth is 0.023, 0.038 and 0.064 when λ takes values of 0.25, 0.5 and 0.74, respectively.

Therefore, the positive externalities dominate over the negative ones. However, the difference between the competitive and the socially optimal outcome rises when the value of λ goes up. An increase in λ reduces the redundancy in R&D, and so the negative externality decreases. This rise in λ also reduces the positive knowledge spillover in the R&D process. However, since the monopoly power does not depend on the value of λ, the decreases in the value of λ raise the supremacy of the positive external effects over the negative externality.

We then obtain the same conclusion as Romer (1990). The social optimum can be achieved by subsidizing the R&D activity. In other words, the government can decentralize the socially optimal solution through a subsidy to R&D expenditure.

6. The Econometric Model.

In this section we use this theoretical benchmark to analyze the empirical evidence about the contribution of the R&D-expenditure. More precisely, we find an econometric specification that allows us to contrast the evidence of an R&D-model of endogenous growth where research and development is achieved through the use of output. To do that we will not consider the existence of fiscal policies.

In our model, the production function expressed in growth rates is given by:

$$\frac{\dot{Y}}{Y} = \beta\frac{\dot{H}}{H} + (1-\alpha-\beta)\frac{\dot{L}}{L} + \alpha\frac{\dot{K}}{K} + \eta\frac{\dot{N}}{N}. \tag{20}$$

However, since the state of technology N is not an observable variable, we must find an expression that permits us to approximate it. To that purpose, we use two equations obtained from the theoretical model, which relates the technology growth rate with some observable variables such as the R&D-expenditure. These equations are the free-entry condition for the R&D sector (4), and the growth rate of the patentee's price (15c). After some algebra (See Appendix B for a detailed explanation) we can write the technology growth rate as a linearly approximated function of the R&D-expenditure growth rate and a dynamic factor:

$$\frac{\dot{N}}{N} \approx \frac{\dot{R}}{R} - \frac{1}{1-\lambda}(e_{31} \quad -e_{31})\begin{pmatrix} 1 \\ e_P \end{pmatrix}(x_0 - \bar{x})\exp(-\theta\, t). \tag{21}$$

In this expression θ denotes the absolute value of the stable eigenvalue of the dynamic system and $(1 \quad e_p)'$ is the normalized eigenvector associated to θ.

Therefore, we can approximate the growth rate of the technology by using a directly observable component, $\dot{R}/R$, which is controlled by economic agents, and a temporal component that includes a constant factor, which depends on the initial level and the steady state value of the ratio capital stock-technology. Thus, if

$$-\eta \frac{1}{1-\lambda}(e_{31} \quad -e_{31})\begin{pmatrix} 1 \\ e_P \end{pmatrix}(x_0 - \bar{x})\exp(-\theta\, t) = a_i \exp(-\theta\, t), \tag{22}$$

we can then transform (20) into

$$\frac{\dot{Y}}{Y} = \beta\frac{\dot{H}}{H} + (1-\alpha-\beta)\frac{\dot{L}}{L} + \alpha\frac{\dot{K}}{K} + \eta\frac{\dot{R}}{R} + a_i \exp(-\theta\, t). \tag{23}$$

The fixed effect a_i is different for each country, for this reason to estimate it we will use dummies to distinguish among seven homogeneous groups of countries.[9]

This equation (23) is estimated using pooled cross-section data for a sample of 21 OECD countries with five observations for each country, which correspond to the time period 1965-1990 at five years intervals. Income, population, labor force and physical capital stock data are taken from the Summers-Heston Penn World Table 5.6. The information about total R&D-expenditure comes from OECD statistic. Data for human capital stocks is obtained from the revised Barro and Lee data set (1996). The proxy used for the human capital is the estimated years of schooling.

[9] These groups are: North-America, Oceania, Japan, the poorest EC countries (Greece, Portugal, Ireland and Spain), the central EC countries the Scandinavian EC countries and the European non-EC countries.

The result obtained from the estimation of equation (23) by Non Linear Squares, can be presented as follows:

$$\frac{\dot{Y}}{Y} = \underset{(2.5)}{0.19}\frac{\dot{H}}{H} + 0.54\frac{\dot{L}}{L} + \underset{(2.6)}{0.27}\frac{\dot{K}}{K} + \underset{(2.2)}{0.08}\frac{\dot{R}}{R} + \exp(\underset{(1.1)}{-0.36}\ t)\sum \hat{a}_i dummies + \varepsilon_t . \qquad (24)$$

Equation (24) shows that the estimated coefficients of human capital, physical capital and R&D-expenditure are positive and significant. Note that t-statistics are in parenthesis below the corresponding coefficients. The t-statistic for the coefficient of the labor force does not appear because we have imposed that this coefficient is equal to 1-α-β.

Concerning R&D-activity, the estimation shows a strong positive correlation between the growth of the total R&D-expenditure and the growth of the GDP. The estimated value is 0.08. Although we directly estimate a structural econometric specification, the magnitude of this coefficient is similar to the previously obtained by Nonneman and Vandhoudt (1996) and Murthy and Chien (1997). They estimate a convergence equation and in both cases their results show that the estimated coefficient of the R&D-variable is 0.09.

On the other hand, we want to note that the sign of the different $\hat{a}_i$'s, which are not presented here, is positive for all groups of countries except for New Zealand and Australia.[10] Therefore, we can conclude that, in general, during the transition to the steady state the ratio K/N is larger than its stationary value.[11] It means that there is an over-accumulation of physical capital with

[10] We obtain seven coefficients, each of them represents the value of this component for each group of countries. The magnitude of these coefficients is very similar, except for Japan and the poorest group of EC countries that are larger. This result means that these groups of countries are in a similar position with respect to their long run equilibrium. However, only the coefficients of Japan and the poorest group of EC countries have significant coefficients.

[11] See Appendix C for a proof of this statement.

respect the state of the technology. In other words, this conclusion reveals that the technology level of these countries is lower with respect to their physical capital stock.

To end with, note that the estimated value of θ gives us an approximated measure of how the distance between the initial values of the variable $x = \ln(K/N)$ and its steady state is reduced. However, in our estimation this coefficient seems to be not significant.

7. Conclusions.

This paper has presented a model where the expenditure in the R&D-activity determines the sustainable growth of output. In this framework, we have considered the existence of a government interested in implementing fiscal policies to stimulate these R&D-investments and thus to encourage economic growth. Unlike preceding semi-endogenous growth models without scale effects, introduced by Jones (1995b), and those in which technological progress shows up as an expansion of the number of varieties of products, these fiscal policies will have growth effects. This issue makes a significant step forward compared to the previous literature. The key of this result is the assumption of a technology of innovation based on R&D-expenditure rather than labor. In particular, we have found that not only the introduction of a subsidy to the R&D-investment will encourage the innovation activity, but also the introduction of a subsidy to physical capital production. Thus, in the environment defined for our model, the physical capital subsidy positively affects the long run growth rate because it provides incentives to increase the variety of capital goods. However, we have numerically found that the growth-maximizing subsidy structure is one where the subsidy to the R&D-expenditure is strictly positive and the subsidy to physical capital is non-positive.

Moreover, the simulation exercise has also allowed us to numerically compared the R&D-expenditure and the growth rate along the BGP that emerge from the equilibrium in absence of fiscal policy with the one that would emerge from the social planning problem. Under the assumptions of the paper, we have found that the market outcome is characterized by an under-investment in R&D.

The paper also has empirically analyzed how R&D-expenditure affects productivity growth. For that purpose, we have made use of an econometric model derived from the theoretical one. More precisely, we have found an expression that can approximate the growth rate of technology through the R&D-expenditure and other dynamical components. At this point, using cross-country data, we have estimated this structural specification. The estimated coefficient corresponding to the R&D-expenditure regressor reveals a strong positive relationship between the growth of total R&D-expenditure and the growth of the GDP. The estimated value of this coefficient tells us that a 1% rise in the aggregate R&D-expenditure will increase the real GDP by 0.08%. One could think that this value is underestimated. If higher R&D leads to higher capital accumulation, then it may have a bigger effect on growth than indicated by the 0.08 coefficient. Future research could be to analyze a simultaneous equation treatment.

APPENDIX

Appendix A. Proof of Proposition 3.

First, we derive the stationary values of X, Z and P_D. From Equation (15c) we get $\overline{X} = \overline{P}_D \,\alpha/(1-\alpha(1-s_k))$. Using the previous equality, we can also express $\overline{Z}$ as a function of $\overline{P}_D$ from (15a). With these stationary values, we get from (15b) the following equation in $\overline{P}_D$:

$$\frac{(1-\tau)\alpha^2}{\sigma}\left(\frac{\alpha\overline{P}_D}{1-\alpha(1-s_k)}\right)^{(\alpha-1)}-\gamma^{1/1-\lambda}\left(\frac{\overline{P}_D}{(1-s_R)}\right)^{(\lambda/1-\lambda)}-\frac{\rho}{\sigma}=0.$$

Then, applying the Implicit Function Theorem to this equation, we calculate the impact of the subsidies to both investments and the income tax on $\overline{P}_D$ as follows

$$\frac{d\overline{P}_D}{ds_R}=-\frac{\delta G(\overline{P}_D)/\delta s_R}{\delta G(\overline{P}_D)/\delta\overline{P}_D}=\frac{\lambda\left(\gamma/(1-s_R)\right)^{1/1-\lambda}\overline{P}_D^{\,(\lambda/1-\lambda)}}{(1-\lambda)D_P}<0, \quad \text{(A1)}$$

$$\frac{d\overline{P}_D}{ds_k}=-\frac{\delta G(\overline{P}_D)/\delta s_k}{\delta G(\overline{P}_D)/\delta P}=\frac{(\alpha-1)\alpha^3(1-\tau)}{\sigma(1-\alpha(1-s_k))D_P}\left(\frac{\alpha\overline{P}_D}{(1-\alpha(1-s_k))}\right)^{(\alpha-1)}>0, \quad \text{(A2)}$$

$$\frac{d\overline{P}_D}{d\tau}=-\frac{\delta G(\overline{P}_D)/\delta\tau}{\delta G(\overline{P}_D)/\delta\overline{P}_D}=\frac{\alpha^2(1-\tau)(1-s_k)^{1-\alpha}}{\sigma D_P}\left(\frac{\alpha}{(1-\alpha)}\overline{P}_D\right)^{(\alpha-1)}<0, \quad \text{(A3)}$$

where

$$D_P=\frac{\delta G(\overline{P}_D)}{\delta\overline{P}_D}=\frac{(\alpha-1)\alpha^2(1-\tau)}{\sigma}\left(\frac{\alpha}{(1-\alpha(1-s_k))}\right)^{(\alpha-1)}\overline{P}_D^{\,(\alpha-2)}$$
$$-\frac{\lambda}{1-\lambda}\gamma^{1/1-\lambda}(1-s_R)^{-\lambda/1-\lambda}\left(\overline{P}_D\right)^{(2\lambda-1/1-\lambda)}<0.$$

Using $\overline{X}=\overline{P}_D\,\alpha/(1-\alpha(1-s_k))$ we compute the impact of fiscal policies on $\overline{X}$ as:

$$\frac{d\overline{X}}{ds_R}=\frac{\alpha}{(1-\alpha)(1-s_k)}\frac{d\overline{P}_D}{ds_R}<0, \quad \text{(A4)}$$

$$\frac{d\overline{X}}{ds_k}=\frac{\alpha}{(1-\alpha(1-s_k))}\frac{d\overline{P}_D}{ds_k}-\frac{\alpha^2}{(1-\alpha(1-s_k))^2}\overline{P}_D$$
$$=\frac{\alpha^2\lambda\gamma^{1/1-\lambda}(1-s_R)^{-\lambda/1-\lambda}\overline{P}_D^{\,2\lambda-1/1-\lambda}}{(1-\alpha(1-s_k))^2(1-\lambda)D_P}<0, \quad \text{(A5)}$$

$$\frac{d\overline{X}}{d\tau}=\frac{\alpha}{(1-\alpha(1-s_k))}\frac{d\overline{P}_D}{d\tau}<0. \quad \text{(A6)}$$

Finally, to analyze the impact of the fiscal policies on $\bar{g}$, we differentiate (17) with respect to s_R, s_k and τ. Hence, using the previous results, we can prove that

$$\frac{d\bar{g}}{ds_R}=\frac{\alpha^2(\alpha-1)(1-\tau)\lambda}{\sigma(1-\lambda)D_P}\left(\frac{\gamma}{(1-s_R)}\right)^{1/(1-\lambda)}\left(\frac{\alpha}{(1-\alpha(1-s_k))}\right)^{\alpha-1}\bar{P}_D^{\,(\alpha-2)+(\lambda/1-\lambda)}>0\,, \tag{A7}$$

$$\frac{d\bar{g}}{ds_k}=\frac{\lambda}{1-\lambda}\left(\frac{\gamma\,\bar{P}_D^{\,(2\lambda-1)}}{(1-s_R)^{\lambda}}\right)^{(1/1-\lambda)}\frac{d\bar{P}_D}{ds_k}>0\,, \tag{A8}$$

$$\frac{d\bar{g}}{d\tau}=\frac{\lambda}{1-\lambda}\left(\frac{\gamma\,\bar{P}_D^{\,(2\lambda-1)}}{(1-s_R)^{\lambda}}\right)^{(1/1-\lambda)}\frac{d\bar{P}_D}{d\tau}<0\,. \tag{A9}$$

Note that from (A2), (A8) and (A7) we can directly obtain that $\dfrac{d\bar{g}/ds_k}{d\bar{g}/ds_R}<1$.

Appendix B. The Econometric Model.

In this Appendix, we will derive expression (23). First, to facilitate the explanation, consider the notation $m_t=R_t/N_t$. Note that (4) implies that $R_t=N_t\left(\gamma P_{Dt}/(1-s_R)\right)^{1/1-\lambda}$. We will use this condition to obtain an approximation for $\dot{N}/N$. Thus, differentiating it with respect to time and using the new notation, we obtain:

$$\frac{\dot{N}}{N}=\frac{\dot{R}}{R}-\frac{1}{1-\lambda}\frac{\dot{P}_D}{P_D}, \quad \text{i.e. in terms of } m: \quad \frac{\dot{m}}{m}=\frac{1}{1-\lambda}\frac{\dot{P}_D}{P_D}=\frac{1}{1-\lambda}\dot{P}. \tag{B1}$$

Hence, we use this relationship to approximate the evolution of technology. From the market equilibrium we observe that the growth rate of technology depends on how much firms increase their expenditure on research and how the price of these new designs rises. Note that the growth rate of the patentee's price is one of the equations of the dynamic system that describes the behavior of the economy. Thus, the second step to obtain the approximation of the growth rate of technology is to solve this system. In particular, we use the policy function of the prices with respect to the transformed variable $x = \ln(K/N)$. This policy function tells us that the patentee's price level is determined by the distance between the initial level and the steady state of the ratio from capital stock to the number of designs. Thus, by using (15c), we can rewrite:

$$\frac{\dot{m}}{m} = \frac{1}{1-\lambda}\left(\alpha^2 e^{(\alpha-1)x} - \alpha(1-\alpha)e^{\alpha x - P}\right) \equiv G(x,P). \tag{B2}$$

At this point, we make the following first order Taylor approximation of (B2):

$$G(x,P) \approx G(\bar{x},\bar{P}) + \begin{pmatrix} G_{\bar{x}} & G_{\bar{P}} \end{pmatrix}\begin{pmatrix} x-\bar{x} \\ P-\bar{P} \end{pmatrix}, \tag{B3}$$

where $\begin{pmatrix} G_{\bar{x}} & G_{\bar{P}} \end{pmatrix}$ is the Jacobian of $G(x,P)$ evaluated in the steady state. We also note that $G(\bar{x},\bar{P})$ is zero and we can write $G(x,P) = \dot{P}/(1-\lambda)$ where we know that e_{32} is zero and $e_{33} = -e_{31}$. Hence, we can rewrite (B3) as:

$$G(x,P) \approx \frac{1}{1-\lambda}\begin{pmatrix} e_{31} & -e_{31} \end{pmatrix}\begin{pmatrix} x-\bar{x} \\ P-\bar{P} \end{pmatrix}. \tag{B4}$$

We also know from the stability properties of Section 4 that the linear approximation of the policy function of P is given by: $(P-\bar{P}) = (x_0 - \bar{x})e_P \exp(-\theta t)$, where θ is the absolute value of the stable eigenvalue of Jacobian matrix, and $e_P = e_{31}/(-\theta + e_{31})$ is the third component of the associated eigenvector. Thus:

$$G(x,P) \approx \frac{1}{1-\lambda}\begin{pmatrix} e_{31} & -e_{31} \end{pmatrix}\begin{pmatrix} (x_0-\bar{x})\exp(-\theta t) \\ (x_0-\bar{x})e_P \exp(-\theta t) \end{pmatrix}. \tag{B5}$$

Using (B2) and (B5), $\dot{m}/m$ can be approximated by

$$\frac{\dot{m}}{m} \approx \frac{1}{1-\lambda}(e_{31} \quad -e_{31})\begin{pmatrix} 1 \\ e_P \end{pmatrix}(x_0-\bar{x})\exp(-\theta t). \tag{B6}$$

Moreover, since $\dot{m}/m = \dot{R}/R - \dot{N}/N$, we can state that

$$\frac{\dot{N}}{N} \approx \frac{\dot{R}}{R} - \frac{1}{1-\lambda}\begin{pmatrix} e_{31} & -e_{31} \end{pmatrix}\begin{pmatrix} 1 \\ e_P \end{pmatrix}(x_0 - \bar{x})\exp(-\theta t). \qquad \text{(B7)}$$

The second part of the right hand side is an expression that depends on time. Therefore we can approximate the growth rate of the technology, using a directly observable component, which is controlled by economic agents, that is the R&D-expenditure growth rate $\dot{R}/R$, and a temporal component, $a_i \exp(-\theta\, t)$. This temporal component includes a constant factor, a_i, which depends on the initial level and the steady state value of the ratio capital stock-technology. Thus, we can obtain equation (23).

Appendix C. The over-accumulation of physical capital during the transition.

We will show that the empirical result of a positive fixed effect implies that there is an over-accumulation of physical capital with respect to the level of technology during the transition to the steady-state equilibrium. In other words, we want to prove the if $a_i > 0$ then $(x - \bar{x}) > 0$. First, we note that:

$$\frac{\dot{m}}{m} \approx \frac{1}{1-\lambda}\begin{pmatrix} e_{31} & -e_{31} \end{pmatrix}\begin{pmatrix} x - \bar{x} \\ P - \bar{P} \end{pmatrix} = \frac{1}{1-\lambda} e_{31}\left((x - \bar{x}) - (P - \bar{P})\right). \qquad \text{(C1)}$$

Furthermore, since the linear approximation around the steady state of the policy function of the P can be written as $(P - \bar{P}) = e_P (x - \bar{x})$, then we transform (C1) into

$$\frac{\dot{m}}{m} \approx \frac{1}{1-\lambda} e_{31}(x - \bar{x})(1 - e_p) = -\frac{1}{1-\lambda}\frac{\theta\, e_{31}}{(-\theta + e_{31})}(x - \bar{x}), \qquad \text{(C2)}$$

where we have used the equality $e_P = e_{31}/(-\theta + e_{31})$. Thus, since $e_{31} < 0$ and $\theta > 0$, then $\dot{m}/m < 0$ if and only if $(x - \bar{x}) > 0$. Therefore, noting from Appendix B that $a_i \exp(-t) = -\eta\, \dot{m}/m$, we prove that $a_i > 0$ implies $(x - \bar{x}) > 0$.

References

Aghion, P. and P. Howitt, 1992, A Model of Growth through Creative Destruction,Econometrica 60 (2), 323-351.

Aghion, P. and P. Howitt, 1998, Endogenous Growth Theory (The MIT Press).

Barro, R. and J. W. Lee, 1993, International Comparisons of Educational Attainment, Journal of Monetary Economics 32, 363-394. (We use data set '96)

Barro, R. and X. Sala-i-Martin, 1995, Economic Growth (New York. McGraw-Hill)

Coe, D. and E. Helpman, 1995, International R&D Spillovers, European Economic Review 39-5, 859-887.

Coe, D., E. Helpman, and A. Hoffmaister, 1997, North-South R&D Spillovers, The Economic Journal 107, 134-149.

De la Fuente, A., 1997, The Empirics of Growth and Convergence: A Selective Review, Journal of Economic Dynamics and Control, 21, 23-37.

Dinopoulos, E. and P. Thompson, 1998, Schumpeterian Growth without Scale Effects, Journal of Economic Growth, 3(4), 313-337.

Dinopoulos, E. and P. Thompson, 1999, Scale Effects in Schumpeterian Models of Growth, Journal of Evolutionary Economics, 9(2), 157-186.

Engelbrecht, H-J, 1997, International R&D Spillovers, Human Capital and Productivity in OECD Economies: An Empirical Investigation, European Economic Review 41, 1479-1488.

Grossman, G. and E. Helpman, 1991, Innovation and Growth in the Global Economy, (MIT Press, Cambridge).

Jones, C., 1995a, Time-Series Tests of Endogenous Growth Models, Quarterly Journal of Economics 110, 495-525.

Jones, C., 1995b, R&D-Based Models of Economic Growth, Journal of Political Economy 103, 759-784.

Kortum, S., 1997, Research, Patenting and Technological Change, Econometrica 65(6), 1389-19.

Mankiw, G., D. Romer and D. Weil, 1992, A Contribution to the Empirics of Economic Growth, Quarterly Journal of Economics 107-2, 407-437.

Murthy, N. R. V. and I. S. Chien, 1997, The Empirics of Economic Growth for OECD Countries: Some New Findings, Economic Letters 55, 425-429.

Nonneman, W. and P. Vandhoudt, 1996, A Further Augmentation of the Solow Model and the Empirics of Economic Growth for OECD Countries, Quarterly Journal of Economics 111, 943-953.

OCDE, Basic Science and Technology Statistics, Paris 1991.

Pecorino, P., 1993, Tax Structure and Growth in a Model with Human Capital, Journal of Public Economics 52, 251-71.

Peretto P., 1998, Technological Change and Population Growth, Journal of Economic Growth, 3(4), 283-311.

Rivera-Batiz, L and P. Romer, 1991, Economic Integration and Endogenous Growth, Quarterly Journal of Economics 106-2, pp. 531-555.

Romer, P., 1990, Endogenous Technological Change, Journal of Political Economy 98-5, 71-102.

Segerstrom P., 1998, Endogenous Growth without Scale Effects, American Economic Review, 88(5), 1290-1310.

Summers, R., and A. Heston, 1996, The Penn World Table: An Expanded Set of International Comparisons, 1050-1093, Quarterly Journal of Economics.

Young, A., 1998, Growth without Scale Effects, Journal of Political Economy, 106(1), 41-63.